ON THE WELL-POSEDNESS OF THE CAUCHY PROBLEM FOR HIGH ORDER ORDINARY LINEAR DIFFERENTIAL EQUATIONS

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On the interval $I = [a, b]$ is discussed for the linear equation of the n-th order

$$u^{(n)} = \sum_{l=1}^{n} p_l(t) u^{(\iota-1)} + p_0(t) , \qquad t \in I \qquad (1)$$

$$u^{i-1}(t_0) = c_{i0}, \qquad (i = 1, \dots, n) \qquad (2)$$

On the well-posedness of the Cauchy problem, i.e. the issue of continuous dependence of the solution on the right side and the initial data.
where $p_l \in L(I; R)$ $(l = 0, \dots, n), t_0 \in I$ and $c_{i0} \in R$ $(i = 1, \dots n)$, I is an arbitrary interval from R. Let u_0 be the unique solution of the Cauchy problem (1),(2).

Along with problem (1),(2) we consider the sequence of problems

$$u^{(n)} = \sum_{l=1}^{n} p_{lk}(t) u^{(\iota-1)} + p_{0k}(t) , \qquad t \in I \qquad (1_k)$$

$$u^{i-1}(t_0) = c_{i0}, \qquad (i = 1, \dots, n) \qquad (2_k)$$

(k=1,2, …), where $p_{lk} \in L(I; R)$ $(l = 0, \dots, n), t_k \in I$ $c_{ik} \in R$ (i=1,…,n; k=1,2, …).

The following issue is investigated in this work: In what sense should the functions $p_{ok}, \dots, p_{nk}$ (k=1,2,…) be close to the functions $p_o, \dots, p_n$, respectively, starting points $t_k (k = 1,2, \dots)$ to t_0 and starting points $c_{ok}, \dots, c_{nk}$ $(k = 1,2, \dots)$ with $c_o, \dots, c_n$ so that the solution u_k of the sequence of Cauchy problems $(1_k), (2_k)$ tends to the solution u_0 of the problem (1),(2) uniformly on segment I, when $k \to +\infty$.

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